LOW ENERGY SPECTRUM OF THE SEA LEVEL ELECTRONS AND MUONS AT 12°N

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ABSTRACT. The slow electron and muon components of cosmic rays at sea level and 12°N geomagnetic latitude have been studied with a multiplate cloud chamber triggered by a coincidence-anti-coincidence system. The differential energy spectra of these electrons have been obtained for energies between (5—300) Mev. The electron spectrum is found to be represented by a simple power law of the form $E^{-1.2}$. In the range interval (7-60) g/cm² of air equivalent the differential range spectrum of muons has also been determined. The spectrum is found to be flat with a mean intensity $(5.89 \pm 0.15) \times 10^{-6}$/g sec. sterad in the range interval (15-60) g/cm.² of air equivalent. Below this range the muon intensity falls off gradually. A comparison of the total intensities of muons and electrons has also been given.

1. INTRODUCTION

This paper contains the final and detailed results of previous investigations on the muon spectrum at Calcutta reported earlier by Basu and Sinha (1956–57) A multiplate cloud chamber was triggered by means of a narrow angle three-fold coincidence system placed above the chamber together with an anticoincidence tray placed just below, in order to photograph all particles which stopped inside the cloud chamber. There were seven ¼″ (5.6 g/cm²) of Cu plates inside the chamber and the total amount of material above the chamber was 6.5 g/cm² of air equivalent Any muon stopping inside the chamber showed more than twice minimum ionisation in the last gap while electrons stopped with minimum ionisation. Fast electrons produced small soft showers all the secondaries of which had to stop inside the chamber.

Thus an estimate could be made of the number of low energy muons and electrons crossing our apparatus in a definite interval of time. The following information were obtained. (1) A fine structure of the differential range spectrum of muons within the range interval 6.5–61.3 g/cm² of air equivalent, (2) The differential energy distribution of electrons in the energy interval (5-300) Mev.

2. EXPERIMENTAL

The main body of the cloud chamber consisted of a glass cylinder with illuminated cylindrical volume of 30 cm diameter and 8 cm deep. It was filled with argon to a pressure of 96 cm of Hg and illuminated by Xenon flash tubes

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placed behind pairs of condensing lenses. The triggering arrangement $C_1C_2C_3$–A is shown in figure 1. During this part of the investigation, there were seven 1/4″ Cu-plates inside the chamber. The solid angle times area of the three-fold counter

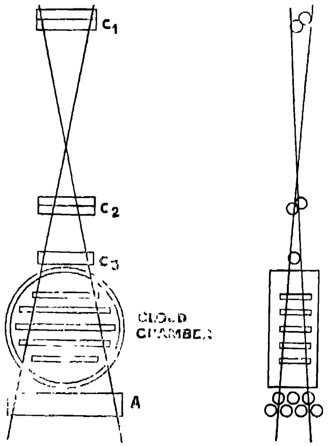

Fig. 1. The experimental arrangement of the various counter trays and the cloud chamber fitted with Cu-plates. In the present experiment there were seven Cu-plates instead of five as shown in the figure.

arrangement was 0.677 cm^2—Steradian. This solid angle was completely covered by the anticoincidence counters placed below the chamber. Stereoscopic photographs were taken and the ionisation and scattering angles of particles (where necessary) were evaluated from both the views.

The apparatus was covered by a thin roof consisting of 0.075 cm thick iron and 3.7 cm celotex Other materials in the solid angle of the counter system were only the counter walls and counter trays All these added together gave a total thickness of 6.5 gm/cm^2 of air equivalent.

The particles stopping in anyone of the seven plates were identified from their ionisation characteristics only. The tracks of stopping electrons were nowhere dense, those of stopping muons were dense in the last compartment and in some cases slightly dense in the previous one and those of stopping protons were dense in all the compartments.

3. ENERGY CALIBRATION OF THE ELECTRONS

The energies of electrons which appeared singly inside the chamber were estimated from their ranges R inside the chamber. The values of R were calculated by numerically integrating the curves of the total rate of energy loss, both radiation and ionisation as a function of energy. The average range of an electron as a function of its energy is given by

$$R = \int_{0}^{E_0} dE/(-dE/dx)$$

where E_0 is the energy of the stopping electron.

Fast electrons almost always give rise to showers and their energies can be estimated from the size of the shower The equation (12) of Rossi (1952) gives the expression for the total number of electrons $\pi(E_0, 0, t)$ as function of thickness t. For electron initiated shower, the maximum value of the function $\pi(E_0, 0, t)$ is given by

$$\Pi_{max}(E_0, 0) = \frac{0.31}{[ln(E_0/\epsilon_0)-0.37]^{\frac{1}{2}}} \cdot \frac{E_0}{\epsilon_0}$$

where,

ϵ_0 = critical energy of the electron in the particular absorber

E_0 = energy of the initiating electron

Π_{max} = number of electrons at shower maximum

Thus from the number of particles at shower maximum, the initial energy of the initiating electron can be estimated, without making any use of information which might be derived from observations at points other than the maxima. This method is not very satisfactory for energies where the maximum number of electrons is small and therefore subject to large fluctuations. Hence an equation given by Bridge, Courant, DeStaebler and Rossi (1954) has been used to estimate the shower energy E_0 According to this equation

$$E_0 = hNt \qquad \ldots \quad (2)$$

where,

h = a constant which depends upon the cut-off energy

N = total number of electron tracks appearing in the separate sections of the chamber through which the shower develops

t = thickness of the plates measured in the direction of the shower axis

Thus Nt is an approximate value for the track length of the observable shower electrons which, according to shower theory, is proportional to the energy of the initiating-particle.

The proportionality constant h was calculated after studying a few well-developed showers with at least 4 secondary electrons at the shower maximum and for which the total number N of electrons at different sections of the cloud chamber was also known. These were consistent with the general theoretical expression for $\pi(E_0, 0, t)$, the maximum of which is given by equation (1). Thus for a number of showers E_0 was found out from equation (1) and for these N and t were also known. In this way a number of values of h was found out and the mean value of $h = 4.2$ Mev/gm of Cu was taken to calculate the initial energy of other shower-initiating electrons from the equation (2).

4. CORRECTIONS OF THE OBSERVED DATA FOR SCATTERING

The most serious error encountered in the measurement of the intensity of the soft component is the multiple scattering of low energy electrons both in air as well as in the apparatus, before the electron enters the cloud chamber. Barker (1955), however, pointed out that the scattering in the roof, the chamber wall, top counters and bottom counters can be neglected even for low energy ($>$ 10Mev) electrons as the number scattered out and scattered in will be approximately the same; but the scattering in the central counters is the main source of error. The electrons which scatter in the top counters in such a way that they go out of the solid angle of the counter system are compensated by those which enter the solid angle only because of the scattering in the top counters, and the third counter tray is so near the chamber that it may be assumed that all the electrons that excite this tray will enter the chamber. But the electrons which scatter in the central counters in such a way that they miss the bottom counter will be completely eliminated and the compensation by others scattered in, will be very small because these are required to pass through the top counters.

Following Barker, we assume that the average path length through two walls of a counter for particles uniformly distributed in space and moving perpendicular to the axis of the counter is equal to πd where d is the thickness of a counter wall. Then assuming the Gaussian angular distribution as given by Rossi and Greisen, (1941) the fraction F of electrons of energy E which are missed on account of scattering will be given by

$$F = 2(\lambda E)^{-2}(2\pi)^{-\frac{1}{2}}[h_2(0)-2h_2(\lambda E)+h_2(2\lambda E)] \quad \ldots \quad (3)$$

where,

$\lambda = 4a(2\pi d)^{-\frac{1}{2}}(21L)^{-1}$

a = diameter of a counter wall = 2.7 cms

L = distance between the top and bottom counters = 60 cms.

d = thickness of a counter wall in radiation lengths = .015 cm Cu = .011 radiation length

Substituting these values we get $\lambda = 0.0345$ Mev^{-1}
$h_2(x)$ are calculated from the tabulated functions

$$h_0(x) = \int_x^\infty \exp(-y^2/2)dy$$

$$h_1(x) = \int^\infty h_0(y)dy$$

$$h_2(x) = \int^\infty h_1(y)dy$$

The values of F are calculated for all the seven energy intervals assuming a mean energy for each energy interval. The corrected number is then obtained by multiplying the observed number of electrons by the factor $(1-F)^{-1}$.

5. ENERGY DISTRIBUTION OF ELECTRONS

Table I shows the distribution of electrons in different energy intervals. One of the characteristic features revealed from this table is that as the electron energy has increased the number of single electrons has decreased and the number of shower initiating electrons has increased. This is in qualitative agreement with shower theory. In column 5 the corrected total number of electrons within the different energy regions has been given. Since the energy *interval* has gradually increased with the increase of energy, the total number of electrons for each energy interval was divided by the corresponding energy interval in order to obtain the energy distribution per Mev in that energy interval.

TABLE I

Energy distribution of electrons

Energy interval Mev	Observed number of electrons Single	shower initiating	Total	Corrected number of electrons	Total number of electrons per Mev
5– 16	108		108	360	32.66±1.09
16– 32	69		69	121	7.55±0.44
32– 56	56	5	61	77	3.20±0.24
56– 87	51	9	60	65	2.11±0.17
87–132	37	7	44	46	1.03±0.09
132–203	29	10	39	40	0.56±0.06
203–310	14	25	39	39	0 37±0.04

This distribution has been plotted in a double log scale in figure 2. It is found that within the energy band (5-300) Mev, the differential energy distribution is well represented by a simple power law of the form $E^{-1.2}$. Barker obtained a distribution of slope -1 between 30 –400 Mev at 45°N, and for energy band above 400 Mev the slope was -1.5.

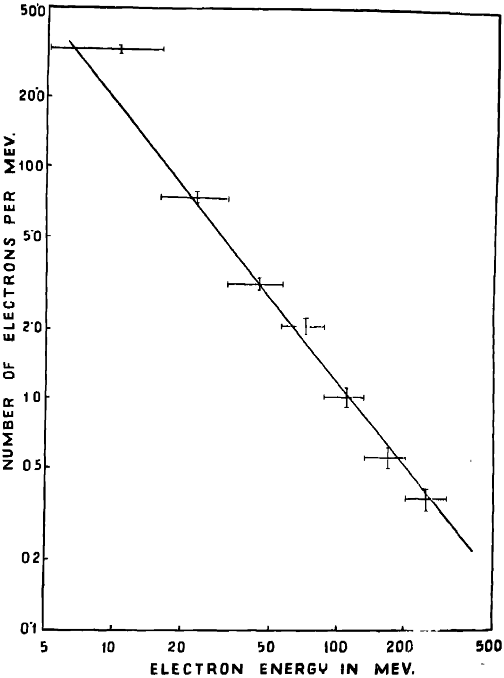

Fig. 2. The differential energy distribution of electrons.

6. RANGE DISTRIBUTION OF MUONS

A differential range spectrum of muons has been obtained previously by the authors, Basu and Sinha (1956-57) in the range interval 6.5–140 g/cm^2 of air equivalent. It was pointed out there, that this range spectrum could not be evaluated accurately at the lowest range interval as the electron contribution in this region cannot be calculated from shower theory. Hence the Cu-plates were split into halves and the muons were experimentally separated from the electrons by ionisation characteristics alone. Furthermore, the splitting of the Cu-plates led to a determination of the fine structure of the muon spectrum in the lowest range interval.

The experiment was divided into two parts. The first part was carried out with no absorber above the chamber and during the second part a 4.5 cm lead absorber was placed above the chamber. The correction for the loss of particles due to scattering in this absorber and in the plates inside the chamber was taken into account exactly in the same manner as explained in the previous paper Basu and Sinha (1956–57). Table II shows the values of the muon intensity I_v. The small gap between (32–36) g/cm² of air is due to the fact that the muons stopping in the last plate were not taken into account because of the inefficiency of the system in photographing these particles due to the emission of decay electrons in the downward direction so as to trip the anti counters.

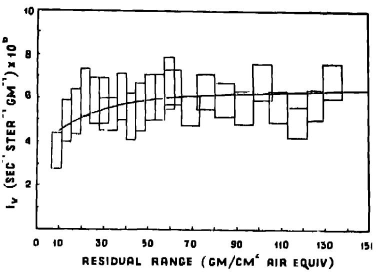

Fig. 3. The differential range spectrum of mu-mesons. The solid curve shown along is Sand's computed sea-level spectrum for mu-mesons.

In figure 3 the combined differential range spectra as found in this experiment and the previous one have been plotted as a histogram. The solid curve shown along the experimental histograms is Sand's (1949) computed sea level range spectrum of muons. The two curves are found to agree satisfactorily within experimental errors. The data below 20 g/cm² of air equivalent have been obtained for the first time in this experiment. It is clear from the histogram that the decrease in the muon intensity when we go below 20 g/cm² as predicted by Sand's computations is real.

7 8. The ratio of electrons to muons for different kinetic energy intervals

From the foregoing data on the intensity of low energy electrons and low energy mu-mesons we can construct a comparative data on the ratio of the intensities of these two kinds of particles at different energy intervals. Hence the whole muon distribution obtained during this and the previous experiments was converted into corresponding kinetic energy intervals from the curves of Rossi (1948). The mu-meson spectrum thus obtained covered the kinetic energy interval (35–304) Mev. In Table III the range as well as the K.E. distribution of mu-mesons have been shown.

TABLE II

Absolute differential range spectrum of muons at 12°N (Fine Structure)

Range interval (g/cm^2 of air equivalent) R	Observed number of muons N	Total sensitive time in hours T	Vertical differential muon-intensity I_v gm^{-1} sec^{-1} $sterad^{-1}$ $\times 10^6$
6.50–10 75	8	215	3 59±0.84
10 75–15 00	11	215	4 94±0.97
15.00–19.25	12	215	5.39±1.03
19.25–23 50	14	215	6.28±1.10
23 50–27 75	13	215	5.83±1 08
27 75–32.00	13	215	5.83±1.08
35 80–40 05	13.7	215	6.05±1.09
40.05 44 30	11.7	215	5.15±1 02
44.30–48.55	12.7	215	5 60±1.07
48.55–52 80	13.7	215	6 05±1.09
52 80–57 05	13 7	215	6.05±1 09
57 05–61 30	14.2	215.	6.72±1 21

TABLE III

Range and kinetic energy distribution of mu-mesons

Range interval (g/cm^2 of air equivalent)	Kinetic energy interval (Mev)	Number of muons	Number of muons per Mev in 215 hours
6.50– 10.75	35– 48	8	0 62±0.15
10.75– 19 25	48– 69	23	1.07±0 15
19.25– 32.00	69– 98	40	1.38±0 13
35.80– 48 55	107–132	37	1 45±0.16
48.55– 61.03	132–158	42	1.63±0.16
72–00– 89.50	181–214	54	1.70±0.15
89.50–106.50	214–246	55	1.70±0.16
104.65–137.70	240–304	102	1.60±0.11

In figure 4 both mu-mesons and electron distributions have been plotted in a bilogarithmic scale where the abscissa are the kinetic energies in Mev and the ordinates the corresponding numbers of particles per Mev. The error has been

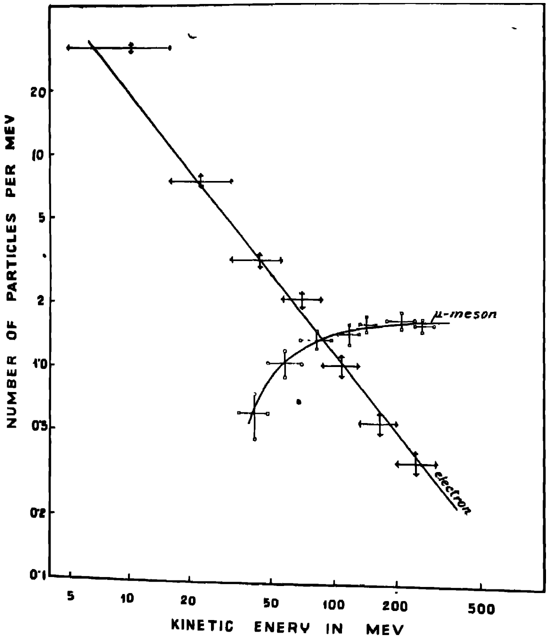

Fig. 4. The differential energy distribution of electrons and muons as function of their kinetic energies.

calculated from the probable statistical error ($0.67\sqrt{n}$) where n is the number of particles. These curves give directly the ratio of electrons to muons within different kinetic energy intervals ranging from 35 to 300 Mev at sea level.

It is interesting to note that the two curves intersect at 95 Mev and therefore electrons and muons of kinetic energy 95 Mev are almost equally abundant at this place.

8. TOTAL VERTICAL INTENSITIES OF THE PENETRATING PARTICLES AND THE LOW ENERGY ELECTRONS

The three-fold counting rate with 10 cm pb above the cloud chamber was found to be 0.317 min^{-1}. A large number of photographs triggered by a three-fold coincidence only, was taken and finally the corrected vertical intensity of the penetrating particles was obtained from the observed rate of accetable pictures, after excluding blanks, showers and heavy particles. This was found to be (0.0073 ± 0.00026) cm^{-2}. sterad^{-1} sec^{-1}. Chandrasekharan *et al* (1950) measured the verticle intensity of penetrating cosmic rays (with 10 cmPb) at Poona (9°N) which is 555 meters above sea level, in terms of counts per minute. From the given geometry of the quadruple coincidence counter telescope used in this experiment, the vertical intensity of penetrating particles at Poona has been found to be 0.0076 cm^{-2} sterad^{-1} sec^{-1}. This is in good agreement with our observations at 12°N at sea-level. The value quoted by Rossi (1948) at 45°N as determined with counter telescope was 0 0083 at sea level.

The vertical intensity of electrons I_s with energy between 5–310 Mev was found to be (0.00143 ± 0.00021) cm^{-2} sec^{-1} sterad^{-1} after correcting for scattering. Combining this value with the total intensity I_v of penetrating particles (0.0073 ± 0.00026), we get a ratio $I_s/I_v = 0.020 \pm 0.03$ for electrons of energy below 300 Mev. at 12°N.

From the data of Hazen (1945) and more recently of Barker (1955) we find that, for latitude $>45°$ the ratio I_s/I_v when I_s expresses electron intensity of energy >300 Mev is nearly 3% Taking for granted that this ratio remains the same at low latitudes we find $I_s/I_v = 0.23 \pm 0.04$ when I_s expresses the intensity of all electrons of energy > 5 Mev. This gives the vertical intensity of electrons at Calcutta (sea-level, 12°N) as (0.00167 ± 0.00029) compared with the value of 0.0022 obtained by Barkar after extrapolating Rossi's curve The ratio I_s/I_v as obtained by Barkar (1955) for Ann arbor (45°N) is 0.311 ± 0.028 with $E > 10$ Mev. Conversly, if we accept that the ratio I_s/I_v remains the same for all latitudes our results would then indicate that the percentage of high energy electrons ($>$ 300 Mev) in the sea level electron spectrum at 12°N is higher than that at 45°N by a factor of about three. An independent measurement of the intensity of electrons of energy >300 Mev at this station is needed to decide between the two alternatives.

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